

GLOBULAR CLUSTER SYSTEMS IN GALAXIES, NEAR AND FAR

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ABSTRACT

Massive star clusters connect with a huge range of astrophysical areas: galactic structures and galaxy mass profiles; the early assembly history of galaxies; star formation under extreme conditions; stellar dynamics; stellar evolution and variability; chemical evolution of metal-poor stars; intermediate-mass black holes. Outstanding prospects for the next decade or more in globular research include:

- Continued advances in computational simulations will allow us to track the cradle-to-grave evolution of massive star clusters, from star formation through early rapid mass loss and on to their final dynamical dissolution.
- The new imaging capabilities offered by ALMA (sub-mm) and JWST (infrared) should allow us to probe the sites of truly massive star-cluster formation in merging and starburst galaxies at unprecedented resolution and to find out just how complex and rapid this process is. Going beyond to higher redshift (out to $z \sim 1$), we may begin to directly see the many “young massive clusters” ($10^6 - 10^8 M_\odot$) formed during the main late stages of large-galaxy formation.
- TMT-class and 8m spectrographs will open the door to building large spectroscopic databases of globular cluster populations in nearby galaxies; GC distribution functions by age and chemical abundance; and new spectral synthesis modelling. Chemical tagging at high spectral resolution holds the potential to track individual SN enrichment histories, and to determine the fraction of field halo stars originating from disrupted GC-like hosts.
- Multiple stellar populations within massive globular clusters have recently been uncovered. Their origins point to complex patterns of age and abundances that are just beginning to be probed.
- Radial-velocity and proper-motion databases for nearby clusters can be tremendously expanded, with 8m and TMT-class spectrographs and with imaging from HST, JWST, LSST, and potentially ngCFHT and a wide-field space-based imager. The resulting analyses of internal kinematics and dynamics will test for the existence of intermediate-mass black holes, dark matter, and the dynamical behavior near the cluster tidal boundary.

Subject headings: globular clusters, galaxy formation

1. EVOLUTION, BEGINNING TO END

The investigation of globular clusters (GCs) has long been one of the areas Canadian astronomy is famous for. Canadians played major roles in the understanding of GC variable stars; ultra-deep photometry of their stellar content with CCD cameras and (later) HST; GC populations in the Local Group and more distant galaxies; stellar evolution codes; and chemical abundance studies. Once regarded as rather simple, monolithic objects, GCs are now diverse laboratories for stellar physics, probes for galaxy formation, and unique windows into star formation under the densest and most extreme conditions.

The “classic” GCs are old (~ 12 Gyr), massive ($\sim 10^4 - 10^7 M_\odot$), compact ($r_{eff} \simeq 2 - 3$ pc) stellar systems virtually free of dark matter (Figure 1). Thousands of GCs are found in the biggest galaxies (Figure 2), marking out symmetric spatial distributions that can extend outwards of $R_{gc} \gtrsim 100$ kpc. GCs are known to fall conveniently into rather distinct metal-poor and metal-rich subpopulations differing by about 1.0 dex in mean metallicity (Figure 3) and ~ 2 Gyr in age.

The old GCs *must* be the survivors of a rather dangerous and violent phase of formation, as well as the subsequent long dynamical evolution within the tidal field of their host galaxy. Most stars are thought to form in the clumpy sites of star formation within giant molecu-

lar clouds (e.g. Lada & Lada 2003; de Grijs 2010), but most of these nascent “clusters” quickly disrupt due to low star formation efficiency and early mass loss driven by stellar winds and SNe. After that, most of the surviving clusters $\lesssim 10^5 M_\odot$ evaporate dynamically within a few Gyr (e.g. Vesperini & Zepf 2003; McLaughlin & Fall 2008, among many others). In addition, the early rapid mass loss phase over the first ~ 30 Myr produces a stochastic increase in cluster effective radius, perhaps responsible for generating the characteristic linear size distribution of the old clusters (Figure 4).

These evolutionary steps imply that the GCs we see *today* are of order *ten times less massive* than their gaseous progenitors (e.g., a normal $10^6 M_\odot$ GC like 47 Tucanae originated from a gas cloud of $10^7 M_\odot$ packed within a radius of one or two parsecs!). This mode of star formation is at a different level than we have successfully modelled in any detail or have easily been able to observe *in situ*. But recent observations of active galaxies at redshifts $z \gtrsim 2$ indicate that a high fraction of all star formation may have happened in these dense, massive regions within protogalactic disks (Elmegreen et al. 2009; Shapiro et al. 2010; Murray & Rahman 2010), so their global importance is far-reaching.

Individual pieces of this evolutionary sequence have been studied, but we must splice the three major phases

(initial star formation \Rightarrow early rapid mass loss \Rightarrow slow N-body dynamical evolution) into a coherent story. The long-held and exciting vision to build **end-to-end simulations of star cluster evolution** seamlessly covering their history from cradle to grave should finally see major progress in the next decade. SPH/AMR simulations of protoclusters with sufficient resolution to track individual stars *with realistic star formation* can now be done for regions of a few hundred Solar masses (Mashchenko et al. 2008; Bournaud et al. 2008); thus another three cycles of Moore’s law will put us genuinely in the proto-GC regime. For the later, secular stage of pure dynamical evolution, N-body simulations are already at the 10^5 -particle level, putting us close to covering the complete GC range.

On the observational side, seeing the conditions of star formation in the extremely dense and (probably) violent proto-GC conditions has always been hampered by locating young, nearby star clusters and protoclusters (Figure 5) at the GC-type mass and density range (but see Murray & Rahman 2010, for several such candidates in the southern Milky Way). ALMA should give us our first genuine high-resolution look deep into the dust-shrouded cores of nascent GCs in nearby starburst galaxies, at the critical stage where star formation is just beginning. At the same time, new JWST infrared imaging will allow us to probe massive young GCs in difficult environments such as spiral disks and active merging systems at larger distances.

2. CHRONOLOGY OF GALAXY FORMATION

Gains in computational power will also be a key to understanding **the formation of massive star clusters in the environment of their host galaxies**. What governs the formation efficiency of massive clusters, in key sites such as the pregalactic dwarfs, gas-rich mergers, or the first few Gyr of hierarchical merging? What is the physical cause for the near-universal phenomenon of bimodality in the GC metallicity distribution? And, what do several thousand ultra-luminous, supermassive protoclusters do to the early global evolution of their host galaxy? Pioneering attempts for Milky-Way-sized systems (e.g. Kravtsov & Gnedin 2005; Prieto & Gnedin 2008) can resolve proto-GCs at the level of only a few particles and do not get important features of GC formation right. Further steps will require resolutions equivalent to particle masses of $10^2 - 10^3 M_\odot$, i.e. 1000 or more particles per proto-GC, with realistic gas physics and star formation prescriptions (cf. Griffen et al. 2010). Advances in supercomputing over the next decade should put such high-resolution simulations for bigger galaxies within reach and will lead to a host of testable predictions.

The **age distribution function** (ADF) for the GCs in a given galaxy can directly constrain the timescales and rates in hierarchical-merging formation models. For more than half a century, the Milky Way GCs have stood as the most fundamental way to test the age of the universe directly through stellar evolution, and to place stringent absolute limits on the redshift range of galaxy formation. The two ways to do this are main-sequence turnoff isochrone fitting, and the cooling times of white dwarfs (Fig. 1). These two methods rely on *different* stellar physics and are thus nearly independent.

Both are now about equally precise (± 1 Gyr) but may be improved to an “ultimate” level of ± 0.3 Gyr by deeper precise photometry, by continued improvements in the stellar models, their transformation into the relevant observational planes, and their chemical abundances. New data from JWST, TMT, LSST, or a wide-field imaging space telescope, would tremendously boost this work.

Age measurement for the GCs in other galaxies is inevitably less precise, but can be done to $\pm 1 - 2$ Gyr through their integrated spectra with combinations of line-strength indices that are variously age- and abundance-sensitive (e.g. Puzia et al. 2005; Woodley et al. 2010a). Promising new routes that employ higher dispersion and much more precise line indices that will go beyond the older Lick-index system are being developed (Colucci et al. 2009; McWilliam & Bernstein 2008) and need coupling to improvements in stellar population synthesis. A TMT-class telescope with a multi-object spectrograph at moderately high resolution would be a major driver for such work, allowing us to extend it to GCs in the galaxies in Virgo, Fornax, and beyond.

Could we actually see parts of the major GC formation eras in action? Existing ADFs and simulations, along with the evidence from $z \sim 2$ disks (e.g. Elmegreen et al. 2009; Shapiro et al. 2010; Murray & Rahman 2010, among many), indicate that many of the *metal-rich* GCs in large galaxies formed only in the past few Gyr, especially during the last few major mergers. Because massive GCs $\sim 10^7$ yr old can have luminosities $M_V < -15$, nearly as bright as supernovae, these should be *directly* visible out to $z \sim 1$ with JWST, opening up remarkably direct potential tests. By contrast, the *metal-poor* GCs originated earlier ($z > 5$, perhaps largely in the pregalactic dwarfs at the beginning of hierarchical merging, or in metal-poor satellites that were later accreted; see Harris & Pudritz 1994).

3. WHENCE MULTIPLE POPULATIONS?

GCs are *not* the ideal “Single Stellar Populations” they were once thought to be. New photometry and spectroscopy shows that the most massive ones may exhibit multiple subpopulations (see Figure 6) and a range of **chemical abundance patterns** (e.g. Piotto 2010). These are suggested to be due to internal dispersions in age, or metallicity, or even helium abundance; and furthermore, the same solution does not seem to work for all such clusters. Evidence from the most massive LMC clusters shows similar anomalies in much younger systems. This is now a large and fascinating problem area being intensively pursued by several groups.

New questions abound. Do higher-mass protoclusters have more complex and longer formation times? How common was later gas infall and distinct second- and third-generation star formation? How distinct are these detailed abundance patterns from field-halo stars and the smallest dwarf galaxies? Various heavy-element abundance anomalies are now mooted to be due to AGB stars; is this plausible and do we really understand late stages of evolution? Do the halo (metal-poor) and bulge (metal-rich) GCs have sites of origin that can be clearly distinguished by chemical tagging? How far can we go to reconstruct the numbers and types of accreted dwarf satellites that still show kinematic and chemical tags in the halo? Are the most massive GCs really remnant nu-

clei of disrupted dwarfs, or can we have different but convergent routes for building them?

This field will be rich ground for the 8m and TMT-class spectrographs through large samples of detailed abundance patterns, not just in the Milky Way but in other nearby galaxies. **Large spectroscopic databases** of GCs in nearby galaxies – both individual stars in the Local Group GCs, and integrated GC spectra in more distant systems – can be built up to rival the photometric databases that we currently have.

New theory will be equally important. For stellar evolution, new grids of models for the late stages of evolution (HB and AGB) are badly needed, as are more advanced model atmospheres to put spectral synthesis on a reliable footing. The recent observations we already have also further reinforce the need (see Section 1) to do high-resolution simulations of massive clusters in formation, with the ability to trace self-consistent internal *and differential* self-enrichment (Bailin & Harris 2009).

4. STRUCTURES AND DYNAMICS

Massive star clusters might, or might not, be the hosts of **intermediate-mass black holes** (IMBH), which in turn *could* have been the seeds from which supermassive black holes grew at the centers of giant galaxies. The evidence for the presence of $\sim 10^4 M_\odot$ IMBHs, as opposed to other alternatives such as radially dependent anisotropy, is probably the best for ω Cen and M31-G1 (Noyola et al. 2008; Gebhardt et al. 2005), but even here the cases are not unequivocal (cf. Anderson & van der Marel 2010). In the next several years, we may be able to build convincing evidence for IMBHs rather than simply testing consistency with a preconception. Critically helpful data will be increased samples of stellar radial velocities deep in cluster cores, and especially space-based proper motion data for the same stars, which will directly address the anisotropy question.

Far beyond just looking for IMBHs, new and extensive proper-motion databases covering the entire radial range within GCs coupled with equally large and precise radial velocity samples will transform the study of their **internal kinematics and dynamics**. A small sample of the remarkable power of this kind of data is illustrated in Figure 7 (e.g. McLaughlin et al. 2006; Anderson & van der Marel 2010). What happens to the higher-energy stars near the nominal “tidal radius” or escape velocity – along with their escape rates in a realistic active tidal field, and the true shape of the cluster profile at and just beyond the tidal radius – are still open questions. During the next decade, N-body simulations should reach a production-level stage of maturity in which the long-term dynamical evolution can be explored in the parameter space of cluster mass, metallicity, and binary fraction.

Such work will also place stringent limits on the absence (or presence) of **dark matter** within GCs, a long-standing question. Comprehensive new solutions for the internal cluster dynamics can be coupled with a complete census of their stellar content all the way past the H-burning limit. We already have evidence that the mass-to-light ratio is rather uniform at $(M/L_V) \simeq 2$ for most GCs but begins to increase systematically for $M > 10^6 M_\odot$ up to levels $(M/L) \sim 4-5$ that approach conventional dE galaxies and Ultra-Compact Dwarfs (UCDs) (Figure 8). How much of this is due only to a (so-far un-

observed) changing mix of stellar population, and how much to their supposed history of origin as nuclei of disrupted dwarfs with their cargo of dark matter? How does the mass and density of the protocluster determine the full shape of the stellar IMF?

Globular clusters mark out a surprisingly narrow *fundamental plane* of structural parameters (e.g. McLaughlin 2000; Barmby et al. 2007; Harris et al. 2010) which was long thought to be fundamentally separate from similarly old small galaxies (dE’s and dwarf spheroidals). However, the discoveries of UCDs (Drinkwater et al. 2003) and the extremely faint dwarf satellites around the Milky Way and M31 (Belokurov et al. 2007; McConnachie et al. 2009) over the past few years, along with measurements of supermassive GCs up to the $\sim 10^7 M_\odot$ regime (Rejkuba et al. 2007; Evstigneeva et al. 2008) have opened up bridges between these various old stellar systems and once again raise the question of **what distinguishes a massive star cluster from a small galaxy**. This landscape is still being explored observationally, but the next decade should see the onset of dynamical models with and without dark matter tuned to their structures. To constrain these, we need considerably more measurements of velocity dispersions, structural profiles, and metallicities for these transition objects, projects well suited to spectroscopy with 8m and TMT-class telescopes as well as space-based imaging (HST, JWST, or a wide-field facility).

Lastly, GCs are near-ideal test particles for tracing the structures and dynamics of galactic halos. GC velocity measurements and spatial distributions provide a powerful way to extend the **galaxy mass profile and dark-matter potential** outward far past where integrated-light work is of any use (e.g. Côté et al. 2001; Woodley et al. 2010b; Schuberth et al. 2010, and see Figure 9). With both the 8m and TMT spectrographs, much more of this can be done. With velocity databases of more than 10^3 particles per galaxy we can begin to explore such questions as the correlations with galaxy luminosity, the dependence of anisotropy on galactocentric radius, and even the shape of the dark-matter halo. The outer halos of big galaxies, where the orbital times are several Gyr, should also hold the visible traces of **substructure and accreted satellites**, which will be detectable in the phase space of the GCs.

Summary: Like many other areas of astrophysics, the study of globular clusters has been *data-driven*, with observations often far outrunning theory. Some areas of GC research now have strong, transparent theoretical foundations (e.g., stellar evolution, N-body dynamics), but others (hierarchical merging, the gas dynamics of cluster formation) are far more demanding from a theoretical or computational perspective – they are not easy, and not reducible to analytical formulations with conveniently small sets of parameters that can be artificially “tested” with narrow observing programs. We argue, however, that this situation is precisely one in which huge progress can be made because the field is so open in so many directions. In the ensuing decade, our new instruments are guaranteed to reveal entirely new structures and trends; and we can expect the computational simulations to grow to the point where they will genuinely gain predictive, and not just explanatory, power.

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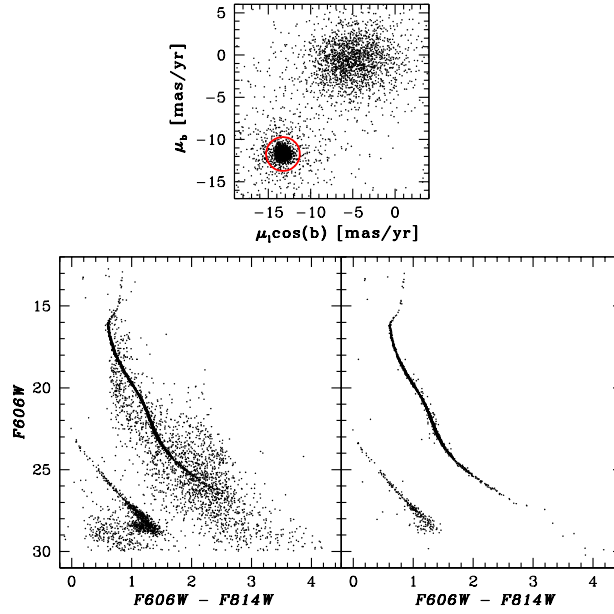


FIG. 1.— Color-magnitude diagram for a sample of stars in the nearby globular cluster NGC 6397, measured from extremely deep HST/ACS photometry. The top panel shows the proper motion vector diagram, with the cluster stars circled. Lower left and right panels show the color-magnitude array before and after removal of proper-motion field stars. Note the deep main sequence of the cluster and white dwarf sequence. The data go fainter than the termination of the H-burning mass range. Figure adapted from Richer et al. (2008).

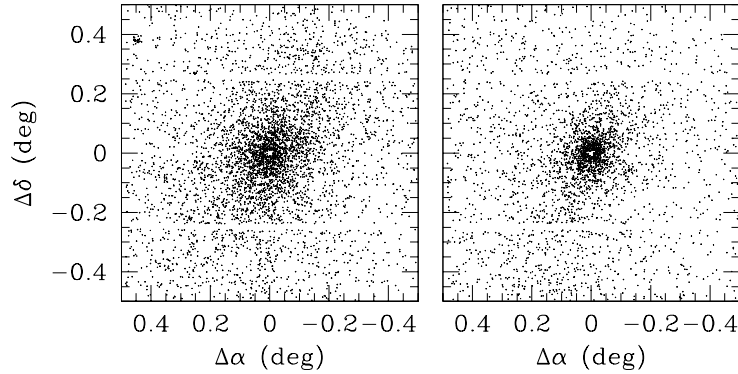


FIG. 2.— Spatial distribution of the bright ($M_i \lesssim -8$) globular clusters around the Virgo giant M87, from photometry with CFHT Megacam. The left panel shows the metal-poor ‘blue’ clusters and the right panel the metal-rich ‘red’ clusters. In most large galaxies, the metal-poor subpopulation is much more spatially extended than the metal-rich one. At the distance of M87, 0.5 degree = 140 kpc. Figure from Harris (2009b).

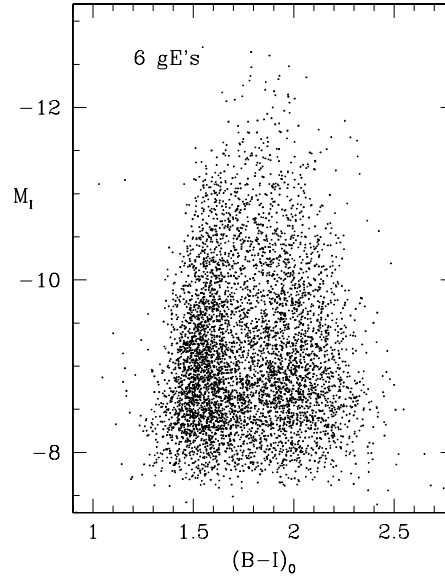


FIG. 3.— Luminosity M_I versus intrinsic color $(B-I)_0$ for a composite sample of 5500 globular clusters in six supergiant ellipticals (all Brightest Cluster Galaxies or BCGs). The ‘blue’ metal-poor sequence is centered at $(B-I)_0 = 1.55$, equivalent to $\langle \text{Fe}/\text{H} \rangle \simeq -1.4$, while the ‘red’ metal-rich sequence is at $(B-I)_0 = 2.0$ or $\langle \text{Fe}/\text{H} \rangle \simeq -0.4$. Figure adapted from Harris (2009a).

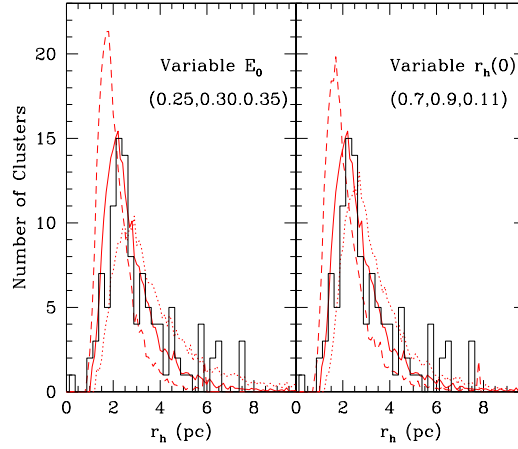


FIG. 4.— Linear size distribution of globular clusters in various galaxies, where r_h is the projected half-light or effective radius. The red curves in each panel are the simulated distributions produced by assuming that the clusters originate from protoclusters with $R_{eff}(init) = 0.9$ pc and then expand due to early rapid mass loss. The left panel shows the distribution for a stochastic range in star formation efficiency of ± 0.05 around a mean $E_0 = 0.3$, while the right panel shows the effect of a stochastic range in initial radius of ± 0.2 pc. Figure drawn from Harris et al. (2010).

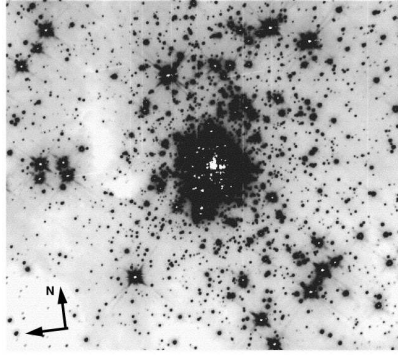


FIG. 5.— HST image in B of R136, the central embedded cluster in 30 Doradus. R136 has several dozen O stars, a total mass of about $50,000M_{\odot}$, and an age of 3 Myr. It is often quoted as an example of a young but small globular cluster; however, if it were located in a bigger galaxy or stronger tidal field, it would probably not survive for a Hubble time. Figure drawn from Sirianni et al. (2000).

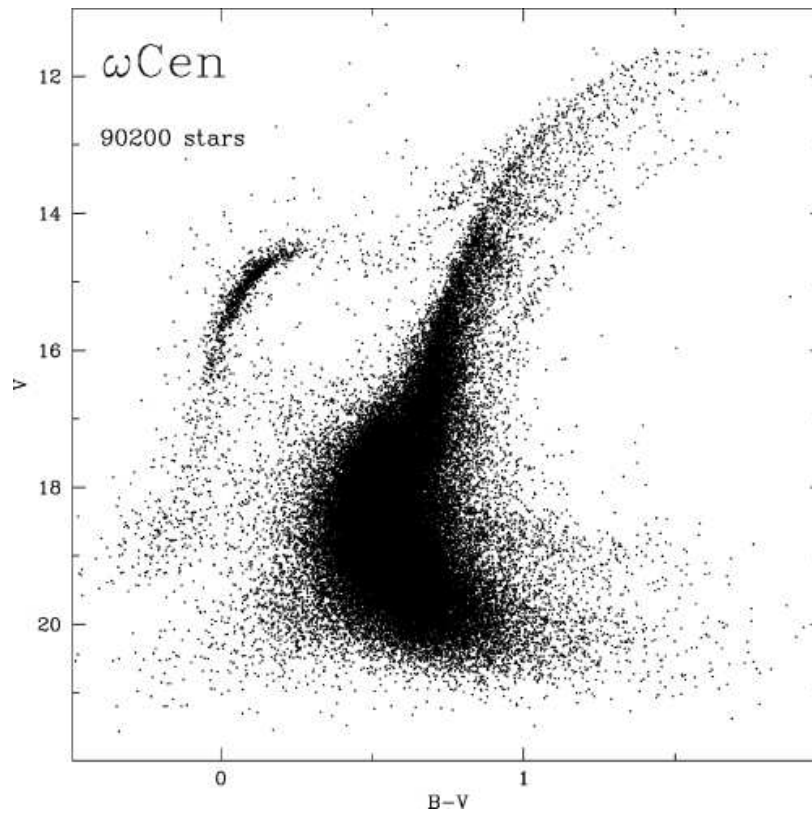


FIG. 6.— Color-magnitude diagrams for the massive, multiple-population cluster ω Centauri. The distinct subpopulations within the cluster show up as the separate turnoff points and red-giant branches. ω Cen is the most massive star cluster in the Milky Way, and has been speculated to be the remnant core of a stripped dE satellite. Figure drawn from Sollima et al. (2005).

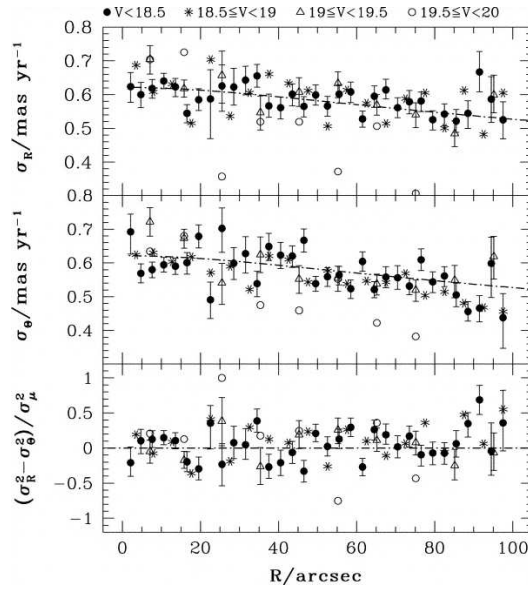


FIG. 7.— Internal velocity dispersion profiles in two dimensions for 47 Tucanae, measured from a sample of 10^4 proper motion stars. The lower panel shows the difference between the two tangential velocity dispersion components, indicating no clear evidence for orbital anisotropy at any radius. Figure drawn from McLaughlin et al. (2006).

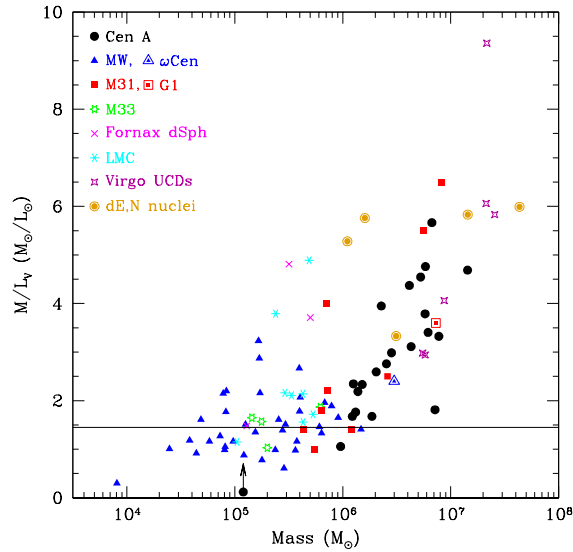


FIG. 8.— Dynamically measured M/L ratios for massive globular clusters, UCDs, dE nuclei, and dwarf spheroidals. Globular clusters less massive than a few $\times 10^5 M_\odot$ (mostly not shown here) have $\langle M/L \rangle \simeq 1.5 - 2$ independent of mass. However, for more massive ones the M/L increases systematically with mass, overlapping with UCDs and dE nuclei. Figure drawn from Rejkuba et al. (2007).

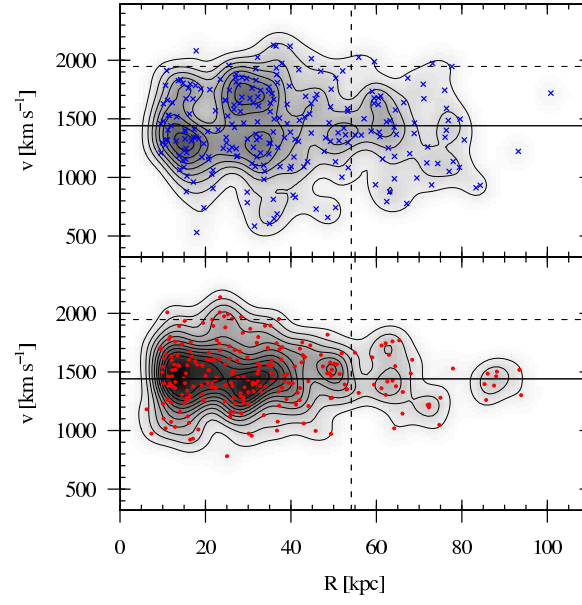


FIG. 9.— Velocity distribution profile for the globular cluster system in NGC 1399, the central Fornax giant. The data sample includes velocities for 700 GCs almost equally divided between metal-poor clusters (top panel) and metal-rich clusters (bottom panel). Samples such as this are used to derive the halo mass profile through the Jeans equation, and can also be used to search for remnant substructure in the outer halo from accreted satellites. Figure drawn from Schubert et al. (2010).